

## Faculty of Science

### M.Sc./M.A. (Mathematics)

#### SYLLABUS FOR M.Sc. ENTRANCE TEST

##### Calculus

Successive differentiation, Derivatives of higher orders, Leibnitz's theorem, Expansion of functions, Maclaurin's theorem and Taylor's theorem, Indeterminate forms, Asymptotes of algebraic curves, angle between radius vector and tangent, derivative of arc length, Curvature, expression for curvature in Cartesian, parametric and polar forms, Radius of curvature in the three forms, Multiple points, convexity and concavity of a curve, Points of inflexion, Tracing of curves (in Cartesian and polar forms), Functions of two or more variables, subtangents and subnormals and their lengths, double points and their types, Beta and Gamma functions,

Integration of the type  $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ , Length of curves, Intrinsic equation of a curve, Volume and Surfaces of solids of revolution of curves given in Cartesian, parametric and polar coordinates.

##### Geometry

2-D, Change of axes and invariants, Pair of straight line through origin, Lines joining the origin to the intersection of a curve and a line, General equation of second degree in two variables, various geometric properties of circle, ellipse, parabola and hyperbola, Intersection of a straight line and a conic section, Equation of a tangent to a conic, Condition of tangency, Pair of tangents from a point, Chord of contact of a pair of tangents to a conic. Pole and polar, Conjugate points and conjugate lines, The condition of conjugacy, Equation of a chord in terms of its middle point, Centre and diameter of a conic, Equation of the conic when the origin is transferred to the centre, Conjugate diameters, Tracing of conic sections, Conjugate hyperbola, Asymptotes, Length and position of axes, Coordinate of foci of ellipse and hyperbolas. Polar equation of a straight line and other conic when focus is at the pole, Directrices, Tracing of the conic  $\frac{x}{a} = 1 + e \cos \theta$ , Equation of Chord and tangent of a conic (in polar form), Normal and Polar for a conic in polar form,

3-D, Direction cosines of a line, Projection of a line segment, Angles between two lines, Distance of a point from a line, Equation of a plane, Length of perpendicular from a point to a plane, General equation of a sphere, Cylinder and its Equation, Equation of a right circular cylinder, Equation of a cone when vertex and guiding curve are known, Equation of a cone when the vertex is at the origin, Condition for general equation of second degree to represent a cone, Tangent plane to a cone and its properties, Condition of tangency, Reciprocal cone, Cone with three mutually perpendicular generators, Number of mutually perpendicular generators.

The standard equation of central conicoids, The tangent plane, Condition of tangency of a plane, Section with a given centre, Locus of the mid-points of a system of parallel chords, Polar plane, Polar lines, Enveloping cone, Classification of central conicoids, Normal, Normals from given point, Conjugate diametral plane and diameters of ellipsoid. Paraboloid (elliptic and hyperbolic). Tangent plane to a paraboloid.

##### Numerical Analysis

Absolute, relative and percentage errors, General error formula, Error in series approximation, Solution of algebraic and transcendental equations, The bisection and Regula falsi method,

Iteration methods namely, Newton-Raphson method, Generalized Newton's method, Solution of system of linear equations using direct methods such as matrix inversion, Gauss elimination and LU decomposition including some iteration methods namely, Jacobi and Gauss-Seidel methods, The algebraic eigen value problems using iterative and Householder methods, Symbols  $\Delta$ ,  $\nabla$ ,  $E$ ,  $E^{-1}$ ,  $D$ ,  $\mu$ ,  $\delta$  and their relations, Newton-Gregory forward and backward difference formulae, Gauss's, Stirling's and Bessel's formulae, Lagrange's formula, Divided differences and their properties, Newton's general interpolation formula, Inverse interpolation formula and Interpolation with cubic splines, Numerical differentiation and integration, Numerical differentiation of tabular and non-tabular functions including error estimations, Numerical integration using Gauss quadrature formulae, Trapezoidal, Simpson's 1/3- and 3/8-rules, Weddle's Rule and Newton-Cotes Formula including error estimation, Curve fitting and approximations, Least squares curve fitting procedures, Different types of approximations, Least squares polynomial approximation, Chebyshev polynomials and its application in various approximations, Ordinary differential equations, Euler's and modified Euler's methods, Picard's method, Taylor series method, Runge-Kutta methods of 2<sup>nd</sup> and 4<sup>th</sup> order, Multi-step methods, Milne-Simpson method, Adam Bashforth-Moulton method, Boundary value problems using Finite difference method.

### **Advanced Calculus**

Functions of several variables, Contour lines, Level curves and level surfaces, Limits and continuity for functions of two and more variables, Partial derivatives, Partial derivatives and continuity, Euler's theorem on homogeneous functions, Differentiability, The chain rule for functions of two and three variables.

Directional derivatives, Gradient vectors, Tangent planes, Linearization and differentials, Extreme values and saddle points, Local and absolute maxima / minima, The method of Lagrange multipliers (with one constraint only), Taylor's series for function of two variables, Partial derivatives with constrained variables, Double integral over a rectangular regions, Double integral as volume, Double integral over non rectangular regions, Fubini's Theorem, Area of a region in a plane, Transformation of double integral from Cartesian to polar co-ordinates and vice-versa, Spherical and cylindrical co-ordinates, Triple integral in Cartesian, Cylindrical and spherical co-ordinates, Line integral, Gradient field, Green's Theorem in a plane, Evaluation of line integral using Green's Theorem, Surface Area, Surface integral, Stokes Theorem.

### **Differential Equations**

Bernouli's equation, Exact differential equations, Integrating factors, change of variables, orthogonal trajectory of curves, Differential equations of first order and higher degree, Equations solvable for  $p$ ,  $y$  and  $x$ , Clairaut's equations, Lagrange's equation, Singular solutions, Homogeneous and non-homogeneous differential equations of order  $n$  with constant coefficients, Complementary functions and particular integrals, Variation of parameters, Linear differential equations with variable coefficients, Reduction of order method, Cauch-Euler and Legendre linear differential equation, Series solution of differential equations, Frobenius method, Total differential equation, condition for integrability, Differential equations of the form:  $Pdx+Qdy+Rdz=0$ , Simultaneous total different equations, Equations of the form  $dx/P=dy/Q=dz/R$ , Methods of grouping and multipliers, Solution of a system of linear differential equation with constant coefficients, An equivalent triangular system, Degenerate case, Laplace transforms, First shifting property, Laplace transforms of some standard functions, Inverse Laplace transforms, Laplace transform of derivatives and integrals. Unit step function and its Laplace transforms, Second shifting property. Unit impulsive function and its Laplace transform. Convolution and periodic function theorems, Solution of linear differential equations with constant coefficients using Laplace transform methods, Definition of a partial differential

equation, Formation of partial differential equations, Equations easily integrable, Classification of first order partial differential equations and their solutions, Solution of quasilinear partial differential equations of first order by Lagrange's method, Nonlinear partial differential equation of first order and their different forms, Charpit's method, Jacobi's Method. Integral surfaces of first order quasilinear partial differential equations through a given curve, Cauchy's problem for first order partial differential equations, Cauchy's method of characteristics, Compatible system of first order partial differential equations, Classification of second order linear partial differential equations, Characteristics and canonical forms for hyperbolic, parabolic and elliptic equations, Solution of quasilinear partial differential equation of second order by Monge's Method. Homogeneous linear partial differential equations of higher order with constant coefficients, Different cases for complimentary functions and particular integrals, Non-homogeneous partial differential equations of higher order with constant coefficients, Reducible and Irreducible equations, Equations reducible to linear partial differential equations with constant coefficients. Method of separation of variables, Fourier series, Even and odd functions and their Fourier series, Change of interval, Half range Fourier sine and cosine series, One dimensional wave equation, one dimensional heat equation, two dimensional Laplace equation and their Fourier series solutions.

### **Linear Algebra**

Binary operations, Group, Ring and Field with examples and elementary properties, Vector spaces, Subspaces, Span of a set, Sum of subspaces, Linear dependence and independence, Dimension and basis, coordinates of a vector relative to an ordered basis, Dimension theorem. Linear transformation and its properties, Range and kernel of a linear transformation, Rank and nullity of a linear transformation, Rank-nullity theorem, Inverse of linear transformation. Vector space  $L(U,V)$  and its dimension, Composition of linear transformations, Matrix associated with a linear transformation, Linear transformation associated with a matrix, Matrix as a linear transformation and its rank and nullity., Elementary row operations and row-reduced echelon form, Inverse of a matrix through elementary row operations, Solution of a system of linear equations, Eigen-values and eigen-vectors.

### **Real Analysis**

Sets, Intervals, Open and closed sets, Bounded and unbounded sets, Supremum and infimum, Neighbourhood of a point, interior points, Open sets and related results, Limit points of a set, Bolzano–Weierstrass Theorem, Closed sets and related results, Countable and uncountable sets, Compact sets and related results, Limit of a function and algebra of limits, Sequential approach, Cauchy's criteria for convergence, notion of continuity, Continuous, Discontinuous functions, Algebra of continuous functions, Properties of continuous functions on closed intervals, Uniform continuous functions and related results, Definitions of derivatives and related results, Increasing and decreasing functions, Darboux's theorem, Rolle's theorem, Mean value theorems of differential calculus and their applications, Taylor's theorem with various forms of remainder, Maclaurin's theorem, Taylor's infinite series, Maclaurin's infinite series expansion of some functions, Functions of bounded variations and their properties, Variation function and related results, Jordan theorem, Vector valued functions, Vector valued functions of bounded variation and related results.

Concept of sequence, Limit points of a sequence, Limit inferior and superior, Convergent and divergent sequences with related theorems, Cauchy's general principle of convergence, Algebra of sequences and statement of related theorems with applications, Monotonic increasing and decreasing sequences.

Introduction of infinite series, Sequence of partial sums and convergence of infinite series, Necessary condition for the convergence of an infinite series, Positive term series, Comparison tests, Cauchy root test, D'Alembert's ratio test with their applications, Alternating series, Leibnitz test, Absolute and conditional convergence, Series of arbitrary terms, Abel's and Dirichlet's tests.

Definition and existence of Riemann integral, Inequalities for Riemann integrals, Refinement of partitions, Darboux's theorem, Sufficient conditions of integrability, Theorems on integrability of the sum, difference, quotient and product of integrable functions, Algebra of Integrable functions, Integrability of the modulus and square of integrable functions, The Riemann integral as a limit of sums, Integrable functions, The fundamental theorem, First and generalized first mean value theorem, Integration by parts, Change of variables, Second mean value theorem.

Definition and existence of the integral, refinement of partitions, Condition of integrability and related results, Integral as a limit of sums and related results, Fourier series, Bessel's inequality, Dirichlet's criteria of convergence of Fourier series, Fourier series for even and odd functions, Fourier series on  $[0, 2\pi]$ ,  $[-1, 1]$  and  $[0, 1]$ , where 1 is a real number.

### **Metric Spaces and Complex Analysis**

Definition and examples of metric spaces, Statements of inequalities of Holder and Minkowski, Properties of metrics, Bounded and unbounded metric spaces, Distance between sets and diameter of a set, Open sets and interior points, Limit points, closed sets and closure of sets, Exterior points, boundary points, boundary of a set, Subspaces.

Convergent and Cauchy sequences in metric spaces, Complete metric spaces, Cantor's intersection theorem, Completion of a metric space, Basis of a metric space, First and second countable metric spaces, Dense sets, Separable metric spaces, Nowhere dense sets, Baire's category theorem.

Open cover, Compact metric spaces, Finite Intersection property, Sequentially compact spaces, Wolzano-Weierstrass property,  $\varepsilon$ -net and totally bounded spaces, Lebesgue's covering lemma, Continuous function between two metric spaces and its characterizations, Continuous functions and compact spaces, Uniform continuous functions, Homeomorphism and equivalent metrics, Uniform convergence, Sequence of functions, Cauchy Criterion for uniform convergence.

Basic concepts, Triangle inequality, Roots of complex numbers, Region in the complex plane, Functions of a complex variable, Algebra of Limits, Limit of a functional a complex variable, Limits involving the point at infinity, Continuity, Derivatives.

Cauchy-Riemann (CR) equations, Sufficient conditions for the differentiability, Polar form of CR equations, Analytic functions, Harmonic functions, Harmonic conjugate, Polar form of Laplace equation, Complex valued functions, Complex line integrals/contour integrals.

Cauchy-Goursat theorem, Consequences of Cauchy-Goursat theorem, Cauchy's integral formula, Cauchy's integral formula for higher order derivatives, Morera's theorem, Cauchy's inequality, Liouville's theorem, Fundamental theorem of algebra, Gauss' mean value theorem, Maximum and minimum modulus theorems/principles.

Power series, Taylor's series, Laurent's series, Singular points, Classifications of singularities, Zeros and poles of order  $m$ , Residues, Calculation of residues, Residue theorem, Evaluation of integrals.

### **Abstract Algebra**

Binary relation, Function, Binary operation, Groups, Various properties of groups, Subgroups, Properties of subgroups, Normal subgroups and important results, Cyclic groups, Generator, Properties of Cyclic groups.

Cosets, Lagrange's theorem, Euler theorem, Fermat's theorem, Isomorphism and homomorphism of groups and related results, Quotient group.

First, Second and Third Isomorphism Theorems, Direct product of groups and its related results.

Permutations, Even and odd permutations, Transportation, Disjoint cycles, permutation groups and its related results, Cayley's theorem, Cauchy's theorem.

Rings, Zero divisors, Integral domains, Division Rings, Field, Subrings and Ideals, Congruence modulo a subring relation in a ring, Simple ring, Algebra of ideals, Ideal generated by a subset, Nilpotent ideals, Nil ideals, Quotient rings, Prime and maximal ideals.

Homomorphism in rings, Natural homomorphism, Kernel of a homomorphism, Fundamental theorem of homomorphism, First and second isomorphism theorems, Field of quotients, Embedding of rings, Ring of endomorphisms of an abelian group.

Factorization in integral domains, Prime and irreducible elements, H.C.F. and L.C.M. of two elements of a ring. Principal ideal domains, Euclidean domains, Uniquefactorisation domains, Different relations between principal ideal domains, Euclidean domains and unique factorization domains.

Polynomials rings, Algebraic and transcendental elements over a ring, Factorization in polynomial ring  $R[x]$ . Division algorithm in  $R[x]$  where  $R$  is a commutative ring with identity, Properties of polynomial ring  $R[x]$  if  $R$  is a field or a U.F.D., Gauss lemma, Gauss Theorem, Eisenstein irreducibility criteria and its applications, Division algorithm for polynomial ring  $F[x]$ , where  $F$  is a field, Reducibility test for polynomials of degree 2 and 3 in  $F[x]$ .

### **Mechanics**

Vector moment of a force, Varignon's Theorem on moments, Resultant of a couple, Resultant of coplanar forces, Equation of line of action of the resultant. Equilibrium of a rigid body, Conditions of equilibrium of three force body. Cables, Suspension and parabolic cables, Intrinsic and Cartesian equation of a catenary, Sag and span, maximum tension in a cable.

Virtual work, Principle of virtual work. Determination of tension in a string and thrust in a rod. Solutions of problems involving equilibrium by principle of virtual work. Centre of gravity. Determination of centre of gravity by integration. Centre of gravity of arcs, plane areas, enclosed areas, solids of revolution and surfaces of revolution.

Kinematics and kinetics, Tangential, normal, radial and transverse components of velocity and acceleration. Motion of projectile without resistance. Projectile motion up and down on inclined plane, Tangent problems, Motion in a resisting medium including projectiles, Upward and downward motion in a resisting medium.

Motion of particles in central orbits, Stability of circular orbits, Kepler's laws of planetary motion, Plane impulsive motion, Direct and oblique impact, Loss of energy during impact, Impact against a fixed plane, Simple harmonic motion, Motion of a simple pendulum.

### **Set Theory, Number Theory and Differential Geometry**

Sets, Cartesian product of sets, Equivalence relation and partition, Fundamental theorem of equivalence relation, Functions, their restrictions and extensions, Characteristic functions and choice functions, Equivalent sets, Infinite sets, Denumerable sets, Countable sets, The continuum, Cardinal arithmetic, Inequalities of cardinal numbers, Cantor theorem.

Schroder Bernstein theorem, Continuum hypothesis, Partially ordered sets, Totally ordered sets, First and last elements, Maximal and minimal elements, Upper and lower bounds, Similar sets and ordered types, Well ordered sets, Transfinite induction, Limit elements, Initial segments, Similarity between well ordered sets and their subsets, Ordinal numbers and their properties.

The division algorithm and derived results, Least common multiple, Greatest common divisor, Euclid's algorithm, prime numbers and related results, Fundamental theorem of arithmetic, Relatively prime integers, Euler's function, Multiplicative arithmetical functions, Mobius functions, Mobius inversion formula.

Congruences, Euler's theorem, Fermat's theorem, order of an integer (mod  $m$ ), Linear congruences, Chinese remainder Theorem, Algebraic congruence (mod  $p$ ), Lagrange's theorem, Wilson's theorem, Algebraic congruence modulo a composite modulus.

Space curves, Plane curves, Parameterization of curves (Generalized and natural parameters) Change of parameter regular curves and singularities. Contact of curves, Contact of a curve and a plane, Frenettrihedron, Osculating plane, Serret-Frenet formulae, Involutives and evolutives, Fundamental Theorem for space curves.

Surface in  $\mathbb{R}^3$ , Implicit and explicit forms of the equation of a surface, Parametric curves on surfaces, Tangent plane, First fundamental form, Angle between two curves on a surface, Area of a surface, Invariance under co-ordinate transformation.

Second fundamental form on a surface, Gauss map and Gaussian curvature, Gauss and Weingarten formulae, Christoffel symbols, Some co-ordinate transformations, Goddard equation and Gauss theorem, Fundamental Theorem of surface Theory.

Curvature of a curve on a surface, Geodesic curvature and normal curvature, Geodesics, Principal directions and lines of curvature, Rodrigue formula. Asymptotic lines, Conjugate Directions, Developable surfaces.