ALIGARH MUSLIM UNIVERSITY, ALIGARH

FACULTY OF SCIENCE

SYLLABUS FOR

M.SC. / M.A. (MATHEMATICS)

Calculus

Improper (infinite) limits; Limits at infinity; Indeterminate forms: L'Hopital rule; Successive differentiation: Leibnitz's theorem; Successive integration; Maclaurin's theorem, Taylor's theorem, Lagrange's and Cauchy's forms of remainders; Hyperbolic differentiation and integration; Improper integrals, Gamma and Beta functions; Reduction formulae for integration; Partial derivatives of functions of two variables, Explicit and implicit representations of plane curves; Tangent and normal to the plane curves; Asymptotes of a curve: Asymptotes parallel to coordinate axes, asymptotes of the general algebraic curves, parallel asymptotes; Curvature of a curve; Evolute of a curve; Convexity, concavity and point of inflexion of a curve; Multiple points: Classification of double points; Tangent at origin; Tracing of Cartesian curves. Arc length of a curve; Arc length function and its derivative; Intrinsic equation of a curve: Deduction of intrinsic equation from Cartesian equation, intrinsic formulae for radius of curvature; Volume of solid of revolution about x-axis and y-axis; Volume of solid of revolution by: cross-section method, disk method, Washer method, and shell method; Surface area of solid of revolution about x-axis and y-axis. Polar co-ordinates and their relations with rectangular co-ordinates; Tangent line to the polar curves; Angle between radius vector and tangent; Perpendicular from the pole on tangent; Determination of the pedal equation of a curve from its Cartesian and polar equations; Asymptotes of polar curves; Radius of curvature in polar and pedal forms; Sketching of polar curves; Area bounded by polar curves; Arc length of a polar curve; Deduction of intrinsic equation of a curve from its polar equation; Volumes and surfaces of solids of revolution in polar forms.

Real valued functions of two and three variables; Limit and Continuity in Cartesian and polar coordinates; Partial derivatives; Mean value theorem; Differentiability; Equality of cross (mixed) derivatives: Young's theorem and Schwartz's theorem; Homogeneous functions and Euler's theorem; Differentials, differentials of higher order; Directional derivative; Gradient and its properties; Taylor's theorem for a function of two variables; Linear and quadratic approximations. Composite functions and Chain rules; Partial derivatives with constrained variables; Implicit functions: Implicit function theorem, derivatives of an implicit function; Jacobian of a transformation: Partial derivatives using Jacobians, inverse of a transformation, Jacobian of an implicit function; Change of variables; Functional dependence. Absolute and relative maxima/minima for a function of two variables; Extreme points, stationary points and saddle points; Extreme values of a function of three variables; Method of Lagrange multipliers; Tangent plane and normal line to a surface; Tangent plane and normal line to a space curve; Family of plane curves/surfaces: Envelopes; Differentiation under the integral sign: Leibnitz's rule. Double integrals

over rectangles and non-rectangular bounded regions: Fubini's theorems; Area and volumes by double integration; Double integrals in polar form; Change in order of integration; Cylindrical and spherical coordinate systems; Triple integrals in rectangular, cylindrical and spherical coordinates; Volumes by triple integrals; Dirichlet's theorem and its Liouville's extension.

Geometry

Pair of straight lines through origin, Homogenous quadratic equations, Angle between two lines, Translation and rotation of axes, Invariants, Removal of cross-product term, Revisit to the concept of conic sections, Translated conics. Analytic descriptions and focal properties of conics, General form of quadratic equations in x & y, Criterion for a pair of straight lines and proper conics, Canonical forms of conics, Centre of a conic, Asymptotes, Conjugate hyperbola, Rectangular hyperbola. Tracing of conics. Tangent, Condition of tangency, Normal, Pair of tangents from an external point, Director circle of a conic, Chord of contact of tangents, Pole and polar, Conjugate points and conjugate lines, Chord in terms of its middle point, Diameter, Equation of circle in diametric form, Conjugate diameters. Parametric equations of a line and conics, Tangent and normal to a conic in parametric form, Polar coordinate system and its relationship with rectangular coordinate system. Polar equation(s) of straight line, circle and conic, Equation of directrix, chord of contact and tangent to a conic in polar forms.

Direction cosines and direction ratios of a line, Projection of a segment, Angles between two lines, Distance of a point from a line, General equation of a plane, Plane through three given points, Equation of a plane in normal and intercept forms, Length of perpendicular from a point to a plane, Angle between planes, Equation of straight line: symmetric and asymmetric forms, Angle between a line and a plane, Line in a plane, Plane through a line, Coplanar lines, Shortest distance between two skew lines. Surface, Pair of planes, Equation of a sphere, Sphere through four given points, Sphere through a given circle, Equation of a sphere in diameter form, Cone, Quadratic cone, Cone with vertex at origin, Right circular cone, Cylindrical surface, Quadratic cylinder, Right circular cylinder, Ellipsoid, Hyperboloid, Elliptic and hyperbolic paraboloids. Condition for general equation of second degree to represent a cone, Intersection of a line with a conicoid, Tangent plane, Condition of tangency, Normal, Number of normals from a given point, Reciprocal cones, Enveloping cylinder and cone, Polar plane, Polar lines. Diameters and diametral planes of a conicoid, Conjugate diameters, Centre of a conic, A general equation of central conicoid, General equation of a paraboloid, Director sphere of a central conicoid.

Space curves, Examples, Plane curves, Parameterization of curves (Generalized and natural parameters), Change of parameter, regular curves and singularities, Contact of curves, Contact of a curve and a plane, Osculating plane, Frenet-trihedron, Curvature and Torsion, Serret- Fernet formulae, Fundamental theorem for space curves. Surfaces in $\mathbb{R}^3$, Implicit and explicit forms of the equation of a surface, Parametric curves on surfaces, Tangent plane, First fundamental form, Angle between two curves on a surface,.Area of a surface, Invariance under co-ordinate transformation. Second fundamental form on a surface, Classification of points on a surface, Gauss map and Gaussian curvature, Gauss and Weingarten formulae, Christoffel symbols, Codazzi equation and Gauss theorem, Fundamental

theorem of surface theory, Co-ordinate transformations. Curvature of a curve on a surface, Geodesic curvature and normal curvature, Geodesics, Principal directions and lines of curvature, Rodrigue formula, Asymptotic lines, Conjugate directions, Euler's theorem.

Numerical Methods and Vector Calculus

Absolute, relative and percentage errors. General error formula. Solution of algebraic and transcendental equations by iteration methods namely. Bisection method. Regula falsi method. Iterative method and Newton-Raphson method. Solution of system of linear equations using LU Decomposition method. Jacobi and Gauss-Siedel method. Symbols Δ, ∇, E, E^{-1}, D, μ, η and their relations. Newton-Gregory interpolation formulae. Forward difference. Backward difference. (derivation of all formulae) Gauss's Forward difference. Gauss Backward difference. Stirling's formulae. Bessel's formulae and Lagrange's interpolation formula. Divided differences and their properties, Newton's general interpolation formula. Numerical differentiation of tabular functions including error estimations. Numerical integration using Gauss quadrature formulae. Trapezoidal, Simpson's 1/3 and 3/8-Rule. Least squares curve fitting procedures and Least squares polynomial approximation. Euler's and modified Euler's methods. Picard's method. Taylor series method. Runge-Kutta Methods of 2^{nd} and 4^{th} order. Solution of boundary value problems of ordinary differential equations using Finite Difference method.

Euclidean space R^n; Open and closed sets in R^n; Limits, continuity, differentiation; Indefinite integration, and definite integration of a vector valued function of single variable, and their physical interpretations; Parametrized curves in plane and space and their properties; Scalar Fields over R^n. Functions from R^n to R^m; Linear functions; Limit, continuity, differentiability, partial and directional derivatives; Jacobian matrix; Chain rule; Mean value theorem; Inverse function theorem; Implicit function theorem. Vector fields in plane and space: Conservative fields and potential functions, curl and divergence, irrotational fields, solenoidal fields; Vector identities; Parametrized surfaces: Tangent plane and normal to a parametrized surface, area of a parametrized surface; Curvilinear co-ordinates system. Line integrals of scalar and vector fields; Fundamental theorem for line integrals; Green's theorem; Applications of line integral; Surface integrals of scalar and vector fields; Orientation of a surface; Stokes' theorem; Gauss divergence theorem; Applications of surface integrals**.**

Set Theory and Number Theory

Revisit of relations and functions, Composition of relations, their matrix representation and properties, Equivalence relation and partition, Fundamental theorem of equivalence relation, cross partition. Functions and their restrictions and extensions, Invertible functions, Characteristic functions and choice functions, Equipotent sets and their properties. Infinite sets, Denumerable sets, Countable sets, Continuum, Cardinals, Cardinal arithmetic, Inequalities of cardinal numbers, Cantor's theorem, Schroeder Bernstein theorem, Continuum hypothesis. Partially ordered sets, Hasse's diagram, Totally ordered sets, Similar sets, and well- ordered sets, First and last elements, Maximal and minimal elements, Zorn's lemma (statement), Axiom of choice, Russel's paradox.

Division algorithm and derived results, Least common multiple, Greatest common divisor, Euclid's algorithm, Prime numbers, Relatively prime integers and related results. Fundamental theorem of arithmetic and its applications, Euler's phi function and its properties multiplicative property, Gauss's theorem and related problems. Congruences, Euler's theorem, Fermat's theorem, Order of an integer (mod m), Linear congruences, Chinese remainder theorem, Primitive roots. Algebraic congruence (mod p), Lagrange's theorem, Wilson theorem, Algebraic congruences with composite modulus.

Differential Equations

Bernoulli equations; Exact differential equations and integrating factors; Change of variables; Orthogonal trajectories of families of curves; First-order equations of higher degree; Equations solvable for *p, y,* and *x*; Lagrange's equation; Clairaut's equation; Singular solutions. Homogeneous and non-homogeneous linear differential equations of higher order with constant coefficients: Complementary functions and particular integrals; Homogeneous and non-homogeneous linear differential equations of higher order with variable coefficients: Cauchy-Euler and Legendre linear differential equations. Solution of homogeneous linear differential equations by inspection; Reduction of order method for homogeneous and non-homogeneous linear differential equations; Elimination of the first derivative term: reduction to normal form; Transformation of the linear differential equations by changing the independent variable. Total differential equations; Condition for integrability; Simultaneous total differential equations: Methods of grouping and use of multipliers; Systems of linear differential equations with constant coefficients; Triangular system of linear differential equations; Degenerate and non- degenerate systems of linear differential equations.

Initial value problems and boundary value problems; Method of successive approximation; Lipschitz's condition; Gronwall's inequality; Picard's theorem; Dependence of solution on initial conditions and on function; Examples of above topics. Power series solutions of second order homogeneous equations; Ordinary points; Regular and irregular singular points; Frobenius' method; Indicial equations; Solutions of Bessel, Legendre, Hermite and Laguerre differential equations; Sturm-Liouville problems; Examples of above topics. Criteria for oscillation and non-oscillation of solutions of certain linear differential equations; Some oscillation theorems such as: Sturm's separation theorem, Sturm's comparison theorem and their applications; Examples of above topics. System of first order linear differential equations; Homogeneous and Diagonalizable Systems; Existence and uniqueness of solution for a system of linear differential equations; First order reduction; Second order reduction; Abel's theorem.

Definition, order and degree of a partial differential equation; Solutions of partial differential equations and their different kinds (complete, general, particular and singular solutions); Classification of partial differential equations; Formation of partial differential equations by elimination of arbitrary constants and arbitrary functions; Equations easily integrable; Method of separation of variables; Examples on above topics. Solution of quasilinear partial differential equations by Lagrange's method; Standard forms of first order partial differential equations, equations

reducible to standard forms; Charpit's method; Cauchy's problem: Integral surfaces through a given curve, method of characteristics; Compatible system of first order partial differential equations; Examples on above topics. Standard forms of second order linear partial differential equations; Classification of semilinear partial differential equations (hyperbolic, parabolic and elliptic equations); Characteristic curves and canonical forms of semilinear partial differential equations; Solution of quasilinear partial differential equations by Monge's method; Examples on above topics. Homogeneous linear partial differential equations with constant coefficients: Reducible and irreducible equations; Nonhomogeneous linear partial differential equations with constant coefficients: Different cases of complementary functions and particular integrals; Cauchy-Euler type and Legendre type linear partial differential equations; Examples on above topics.

Linear Algebra

Binary operations, Definition of field with examples, Definition of vector space with examples, Subspaces, Span of a set, Sum of subspaces, Linear dependence and independence, Basis and dimension of a vector space, Coordinates of a vector relative to an ordered basis, Dimension theorem.
Linear transformation and its properties, Range and kernel of a linear transformation, Rank and nullity of a linear transformation, Rank-nullity Theorem, Inverse of a linear transformation. Vector space L(U,V) and its dimension, Composition of linear transformations, Matrix associated with a linear transformation, Linear transformation associated with a matrix, Rank and nullity of a matrix. Elementary row operations on a matrix, and row echelon form of a matrix, Inverse of a matrix through row operations, Solution of a system of linear equations, Eigen values and Eigen vectors of matrices and linear transformations.

Quotient Space, Change of basis and transition matrices, Vector space Hom (U,V) of linear transformations and its dimension, Algebra A(V), Effect of change of bases on a matrix representation of a linear transformation, Equivalent and similar matrices, Linear functionals, Dual vector spaces, Dual basis and its computation, Second dual spaces, Annihilators, Dual of linear transformations. Inner-product spaces, Normed space, Cauchy-Schwartz inequality, Pythagorean theorem, Polarization identity, Projections, Orthogonal projections, Orthogonal complements, Orthonormality, Matrix representation of inner-products, Gram-Schmidt orthonormalization process, Orthogonal transformation, Inner product space isomorphism, Eigenvalues and eigenvectors of a linear transformation, Characteristic polynomial of a linear operator and related results, Diagonalization of matrices, Invariant subspaces, Cayley-Hamilton Theorem and minimal polynomial of a linear operator, Canonical form, Jordan forms of vector spaces.

Abstract Algebra

Groups and their standard examples including symmetric and dihedral groups and their properties, Order of an element in a group, Subgroups and some basic properties, Centre of a group, Normalizer of a set, Product of two subgroups, Cyclic groups, Generator, its examples and related results. Cosets, Lagrange's theorem and its applications, Index of a subgroup of a group, Euler's theorem, Fermat's theorem, Isomorphism, and homomorphism of groups with examples and related

results, Inner automorphism; Normal subgroups and simple groups. Commutator subgroup and its some basic properties, Quotient groups with examples, First, second and third isomorphism theorems and their applications, Internal and external direct product of groups and their related results, Characterization of a group as a direct product of its two subgroups. Permutations, even and odd permutations, Order of a permutation, Transposition, Cycle, and its length, Disjoint cycles, Permutation groups, Alternating groups and their related results, Signature of a permutation, Cayley's theorem, Cauchy's theorem for finite abelian groups.

Relation of conjugacy, conjugate classes of a group, Number of elements in a conjugate class of an element of a finite group, Class equation in a finite group and related results, Partition of a positive integer, Conjugate classes in Symmetric groups, Sylow's theorems, External and Internal direct products and related results. Structure theory of finite abelian groups, Subgroup generated by a subset of a group, Commutator subgroup of a group, Subnormal series of a group, Refinement of a subnormal series, Length of a subnormal series, Solvable groups and related results, n-th derived subgroup, Upper central and lower central series of a group, Nilpotent groups, Relation between solvable and nilpotent groups, Composition series of a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups.

Rings, Zero divisors, Integral domains, Division rings, Fields, Subrings and ideals, Congruence modulo a subring relation in a ring, Simple ring, Algebra of ideals, Ideal generated by a subset, Nilpotent ideals, Nil ideals, Quotient rings, Prime and maximal ideals. Homomorphism in rings, Natural homomorphism, Kernel of homomorphism, Fundamental theorem of homomorphism, First and second isomorphism theorems, Field of quotients, Embedding of rings, Ring of endomorphisms of an abelian group. Prime and irreducible elements, H.C.F. and L.C.M. of two elements of a ring, Principal ideal domains, Euclidean domains, Unique factorization domains, Different relations between Principal ideal domains, Euclidean domains and Unique factorization domains. Algebraic and transcendental elements over a ring, Polynomial rings over a ring, Factorization in polynomial ring R[x], Division algorithm in R[x], where R is a commutative ring with identity, Properties of polynomial ring R[x] if R is a field or a U.F.D., Gauss lemma, Gauss theorem (statement only), Eisenstein irreducibility criteria and its applications, Division algorithm for polynomial ring F[x], where F is a field, Reducibility test for polynomials of degree 2 and 3 in F[x].

Structure of finite fields, Characterization of finite fields, Automorphisms of finite fields, Factorization of polynomials over finite fields, Factorization of $x^n - 1$, Irreducible polynomials, Constructions of finite fields. Finite extensions and their properties, Degree of extensions, Multiplicative property of degree of extensions, Minimal polynomials, finitely generated extensions, Simple extensions and their properties, Relation between two simple extensions, Quadratic extensions over field of characteristic different from 2, Splitting field and its uniqueness, Description of splitting fields, Algebraic and transcendental elements, Algebraic extensions and their properties, Algebraic closures, Algebraically closed fields and their uniqueness, Fundamental theorem of algebra. Normal extensions and their properties, Roots of unity, Cyclotomic polynomials and extensions, Cyclotomic fields of n^{th} roots of unity, Separable and inseparable polynomials, Separable and inseparable extensions,

Derivations on fields, Prime and perfect fields.

Real Analysis and Complex Analysis

Bounded and unbounded sets, Supremum and infimum, Field axioms, Order axioms and Completeness axioms on $\mathbb{R}$, Nested interval property, Archimedean property of real numbers and representation of real numbers, Denseness property of rational numbers in $\mathbb{R}$, Dedekind theory of real numbers, Dedekind-Cantor axioms, Inequalities of Cauchy-Schwartz, Minkowski and Holder, Extended real number system. Neighbourhood of a point, Interior, exterior and boundary points, Open sets, Limit points and derived set, Bolzano-Weierstrass Theorem, Adherent point and Closure of a set, Closed sets, Compact sets, Heine-Borel Theorem, Connected sets, Dense sets, Perfect sets, Cantor sets. Concept of sequence, Convergent and divergent sequences, Limit inferior and superior, Bounded and unbounded sequences, Monotone sequences, Subsequences, Bolzano Weierstrass theorem for sequence, Cauchy's general principle of convergence, properly divergent sequences, Sequential criterion of closed sets and compact sets, Cantor's theory of real numbers. Introduction to series of real numbers, Sequence of partial sums and convergence of infinite series, Necessary condition for the convergence of an infinite series, Positive term series, Comparison tests (first type and limit form), Cauchy root test, D'Alembert's ratio test with their applications, Alternating series, Leibnitz test, Absolute and conditional convergence, Series of arbitrary terms, Abel's and Dirichlet's tests, Rearrangement of series, Logarithmic and Condensation tests.

Limit of a function; Continuous functions; Sequential criterion for limits and continuity; Types of discontinuities; Properties of continuous functions on closed intervals; Uniform continuous functions; Derivative of a function; Darboux's theorem; Rolle's theorem; Mean value theorems of differential calculus; Taylor's theorem. Definition and existence of Riemann integral; Inequalities for Riemann integrals; Refinement of partitions; Darboux's theorem; Conditions of integrability; Algebra of integrability; modulus and square of integrable functions; Riemann integral as a limit of sums; Classes of Riemann integrable functions; Primitive of a function; Fundamental theorem of calculus; Mean value theorems of integral calculus. Sequence and series of functions; Uniform convergence; Cauchy's Criterion for uniform convergence; Weierstrass M-test; Abel's test; and Dirichlet's test for uniform convergence; Properties of uniformly convergent sequences and series; Continuity, differentiability and integrability for sequences and series of functions; Dini's Theorem; Weierstrass Approximation Theorem. Power series; Radius and interval of convergence; Taylor's theorem; Cauchy's Hadamard theorem; Abel's theorem; Improper integrals and their convergence; Comparison test and Cauchy's test for convergence; Absolute convergence; Abel's Test; Dirichlet's Test; Convergence of Beta and Gamma functions.

Revision of complex number system, Triangle inequality and its applications, Polar and Exponential forms of complex numbers: De-Moivre's Theorem and Euler's formulae, Products and quotients in exponential form, Roots of complex numbers, Point sets and Regions in complex plane, Spherical representation of complex numbers: Stereographic projection. Limits, Continuity and differentiability of functions of a complex variable, Cauchy-Riemann (CR) equations, Sufficient conditions for differentiability, Polar form of CR equations, Analytic functions, Harmonic functions,

Harmonic conjugates, Polar form of Laplace equation, Exponential, Logarithmic, Trigonometric and Hyperbolic functions of complex variables, Complex exponents, Inverse trigonometric and inverse hyperbolic functions. Definite integrals of complex valued functions of a real variable, Contour integrals, Cauchy- Goursat theorem (without proof), Consequences of Cauchy-Goursat theorem, Cauchy's integral formula, Cauchy's integral formula for higher order derivatives, Morera's theorem, Cauchy's inequality, Liouville's theorem, Gauss' mean value theorem, Fundamental theorem of algebra. Sequence and series of complex numbers and their convergences, Sequence and series of complex functions and their convergences, Power series, Absolute and uniform convergence of Power series, Taylor's and, Laurent's series (without proof) with examples, Singular points and their classifications, Zeros and poles of order m, Residues, Calculation of residues, Cauchy Residue theorem.

Metric Spaces and General Topology

Definition and examples of metric spaces, Bounded and unbounded metric spaces, Distance between sets, Diameter of a set, Open and closed balls, Interior points and interior of a set, Open
set, Neighbourhood of a point, Limit point of a set, Closure of a set, Closed set, Boundary points and boundary of a set, Exterior points and exterior of a set, Subspace of a metric space. Sequences in a metric space, Convergent and Cauchy sequences, Complete metric spaces, Relation between completeness and closedness, Cantor Intersection Theorem, Completion Theorem, Countability axioms, Dense sets, Separable spaces, nowhere dense sets, Categories and Baire Category Theorem.
Continuous functions between metric spaces, Sequential criterion for continuous functions, Characterizations of Continuous functions via open and closed sets, Uniform continuous functions, Homeomorphism, Isometry, equivalent metrics. Compact metric spaces, Compact sets and their criterion, Relation between compactness, completeness and closedness, Finite Intersection property, Bolzano-Weierstrass property, Sequential compactness, totally bounded spaces, Continuous functions on compact spaces, separated sets, Connected and disconnected sets, Connected subsets of $\mathbb{R}$, Continuous functions on connected spaces.

Definitions of topology and topological spaces, Examples of topology including discrete topology, indiscrete topology, standard topology on IR, lower limit and upper limit topology, co-finite topology and co-countable topology, Topology induced by a metric, Basis for topology, Subspace topology, K-topology, Order Topology, Product Topology on X×Y, Topology generated by the sub-basis, Closed sets and limit points, Neighbourhoods, Interior, exterior and boundary points, Derived sets, Hausdorff spaces. Continuous functions, Pasting lemma, Homeomorphisms, Convergence in topological spaces, Connected spaces, Connected sets in the real line, Intermediate value theorem, Components and Local connectedness, Path connected, Path components, Locally path connected spaces, Properties of Continuous functions on Connected sets, Compact spaces and their basic properties, Finite intersection property, Compact subspaces of the real line, Extreme value theorem, Lebesgue number, Uniform continuity, Limit point compactness, Sequential compactness, Local compactness, Properties of continuous functions on compact sets. First and second countable spaces, Lindelof spaces, T-1, T-2

(Hausdorff), T-3 (Regular), T-4 (Normal), T-3.5 (Completely regular) spaces and their characterizations and basic properties, Urysohn's lemma, Tietze extension theorem. Product topology (finite and infinite number of spaces), Tychonoff product, Projection maps, Stone Cech Compactification, Comparison of the Box and Product topologies, Quotient topology, Quotient (Identification) spaces with some examples

Functional Analysis

Normed spaces, Banach spaces and their examples, Examples of incomplete normed spaces, Subspace of normed spaces, Isometry on normed spaces, Completion of normed linear spaces, Quotient spaces, Product spaces, Schauder basis, Infinite series in normed space: convergence and absolute convergence, Finite dimensional normed spaces, Equivalent norms, Compactness, Riesz Lemma, Denseness and separability properties. Bounded linear operators and bounded linear functionals with their norms and properties, Unbounded linear operators, Space of bounded linear operators, Dual basis, Algebraic and topological duals and relevant results, Duals of some standard normed spaces, Second duals and canonical embedding, Adjoint operators, Reflexive normed spaces and their properties, Separability of dual space. Inner product spaces and examples, Parallelogram law (without proof), Polarization identity (without proof) and related results, Schwartz inequality (without proof) and triangle inequality, Separability and reflexivity of Hilbert spaces, Orthogonal sets and related results, Projection theorem, Orthonormal sets and sequences, Bessel inequality, Total orthonormal sets and sequences, Parseval relation, Bounded linear functionals on Hilbert spaces: Riesz representation theorem, Hilbert adjoint operators, Self-adjoint operators, Unitary and normal operators. Hahn-Banach theorem and its extended forms, Pointwise and uniform boundedness, Uniform boundedness principle and its applications, Weak convergence of sequences and weak topology in normed space, Open and closed maps, Graph of linear operators and closedness property, Open mapping and closed graph theorems, their consequences and applications.

Differentiable Manifolds

Some important features of calculus of R^n, Topological manifolds, Differentiable atlas, Smooth maps, Diffeomorphism, Equivalent atlases, Differentiable structure on a manifold, Space of smooth maps, Tangent vectors and tangent space, Differential of a smooth map. Vector fields, Commutator of Vector Fields (The Lie Bracket), Lie algebra of vector fields, Integral curve of a vector field, Covectors and Cotangent spaces, Pull back of a linear differential form, One parameter group of transformation, Exponential map, Covariant and Contravariant tensors, Laws of transformation for the components of tensors. Differential forms, Exterior product (the Wedge product), Grassman algebra of forms, Exterior derivative (the Exterior differential operator), Affine Connection, Parallelism, Geodesic, Covariant differentiation of tensors, Torsion and Curvature of a Connection, Structure equation of Cartan, Bianchi's identities. Scalar products on tangent spaces; Space-like, time-like and null vectors; Pseudo Riemannian and Riemannian manifolds; Existence of Riemannian metric on a paracompact manifold; Raising and lowering the index; Levi-Civita connection; Fundamental theorem of Riemannian geometry; Riemannian curvature and its characteristic properties; Ricci and scalar curvature; Einstein manifolds; Divergence identity.