Faculty of Science

M.Sc./M.A. (Mathematics)

SYLLABUS FOR M.Sc. ENTRANCE

Calculus

Successive differentiation: Derivatives of higher order, nth derivative of well-known functions, Leibnitz's theorem (without proof). Expansion of functions, Maclaurin's theorem and Taylor's theorem (without proof), Indeterminate forms: $\frac{0}{0}, \frac{\infty}{\infty}$, 0 x ∞, $\infty - \infty$, and 0^0, 1^∞, ∞^0. Asymptotes of the algebraic curves, parallel asymptotes, Asymptotes parallel to x-axis and y-axis.

Curvature: Angle between radius vector and tangent, derivative of arc length in cartesian, parametric and polar form, Calculation of curvature in cartesian, parametric and polar form. Radius of curvature in the three forms, Multiple points, convexity and concavity of a curve, points of inflexion, Tracing of cartesian curves, Idea of some well known polar curves, Functions of two or more variables, partial differential coefficients, Homogeneous functions, Euler's theorem on homogeneous functions.

Gamma functions, Integration of the type $\int_0^{\frac{\pi}{2}} Sin^m x Cos^n x \, dx$, Length of curves for cartesian, parametric and polar equations, Intrinsic equation from cartesian, parametric and polar equations, Volume and Surfaces of solids of revolution of cartesian, parametric and polar curves.

Geometry

General Equations, pair of straight line through origin, lines joining the origin to the intersection of a curve and a line. The conic section, Equation to a conic section.Intersection of a straight line and a conic.Equation of a tangent to a conic, Condition of tangency.Pair of tangents from a point. Chord of contact of a pair of tangents to a conic. Pole and polar,Conjugate points. Conjugate lines. The condition of conjugacy.Chord with a given middle point. Centre of a conic and diameter. Equation of the conic when the origin is transferred to the centre. Conjugate diameters. Change of rectangular axes without change of origin. Removal of xy-term.Invariants.Nature of conic, Summary.Tracing of Parabola, The asymptotes of hyperbola.Conjugate hyperbola. Asymptotes.The length and position of axes.Coordinate of foci.

Polar equation of a conic when focus is at the pole.Directrices, Tracing of the conic $\frac{l}{r} = 1 + e cos\theta$. Equation of the Chord when the vectorial angles of the extremities are given.Equation of the tangent at the point whose vectorial angle is α. Normal, Polar.Direction cosines of a line.Projection of a segment.Angles between two lines.Distance of a point from a line.Equation of a plane.Length of perpendicular from a point to a plane.Cylinder and its Equation.Right circular cylinder.Equation of a right circular cylinder.

Equation of a cone.Equation of a cone when the vertex is at the origin. Condition for general equation of second degree to represent a cone. Tangent plane to a cone, A property of the tangent plane to a cone. Condition of tangency.Reciprocal cone.Cone with three mutually perpendicular generators, Number of mutually perpendicular generators.

The standard equation of central conicoids.The tangent plane.Condition of tangency of a plane.Section with a given centre.Locus of the mid-points of a system of parallel chords.The Polar plane, Polar lines.Enveloping cone.

Classification of central conicoids.The Normal.Normals from given point. Conjugate diametral plane and diameters of Ellipsoid. Paraboloid (elliptic and hyperbolic).Tangent plane to a paraboloid.

Numerical Analysis

Absolute, relative and percentage errors, General error formula, Error in series approximation, Solution of algebraic and transcendental equations, The bisection and Regulafalsi method, Iteration methods namely, Newton-Raphson method, Generalized Newton's method, Solution of system of linear equations using direct methods such as matrix inversion, Gauss elimination and LU decomposition including some iteration methods namely, Jacobi and Gauss-Siedel methods, The algebraic eigen value problems using iterative and Householder methods.

Symbols of Δ,∇, E, E^{-1}, D, μ and $\delta$and their relations, Newton-Gregory Forward and Backward difference formulae, Gauss's, Stirling's and Bessel's formulae, Lagrange's formula, Divided Differences and their properties, Newton's general interpolation formula, Inverse interpolation formula and Interpolation with cubic splines.

Numerical Differentiation and Integration, Numerical differentiation of tabular and non-tabular functions including error estimations, Numerical integration using Gauss quadrature formulae, Trapezoidal, Simpson's 1/3- and 3/8-Rule, Weddle's Rule and Newton-Cotes Formula including error estimation, Curve fitting and approximations, Least squares curve fitting procedures, Different types of approximations, Least squares polynomial approximation, Chebyshev polynomials and its application in various approximations.

Ordinary differential equations, Euler's and modified Euler's methods, Picard's method, Taylor series method, Runge-Kutta methods of 2^{nd} and 4^{th} order, Multi-step methods, Milne-Simpson method, Adam Bashforth-Moulton method, Boundary value problems using Finite Difference method.

Differential Equations

Exact differential equations.Integrating factors.Total differential equations.Differential equations of first order but not of first degree.Equations solvable for p, y and x. Clairaut's equations.Linear differential equations of order n. Homogeneous and non-homogeneous differential equations of order n with constant coefficients. Complementary functions. Different forms of particular integrals. Linear differential equations with non-constant coefficients.Reduction of order method. The Cauchy-Euler's equation of order n.The Legendre's linear equation.Methods of undetermined coefficients and variation of parameters.Series solution of differential equations.Frobenius method.Different cases.

Bessel, Legendre and hypergeometric differential equations and their solutions.Simultaneous total different equations. Equations of the form dx/P=dy/Q=dz/R. Methods of grouping and multipliers. Solution of a system of linear differential equation with constant coefficients.An equivalent triangular system.Degenerate case.

Laplace transforms. Linearity of Laplace transforms. First shifting property. Table of Laplace transforms. Inverse Laplace transforms. Laplace transform of derivative and integrals. Unit step function and its Laplace transform. Second shifting property. Unit impulsive function and its Laplace transform. Convolution and periodic function theorems.Solution of linear differential equations with constant coefficients using Laplace transform methods.

Formation and solution of a partial differential equations.Equations easily integrable.Linear partial differential equations of first order.Non-linear partial differential equation of first order and their different forms.Charpit's method.

Homogeneous linear partial differential equations with constant coefficients.Different cases for complimentary functions and particular integrals.Non-homogeneous partial differential equations with constant coefficients.Classification of second order linear partial differential equations.

Method of separation of variables.Fourier series.Even and odd functions and their Fourier series. Change of interval. Vibration of stretched string.One and two dimensional heat flow.Solution of Laplace equation in Cartesian and polar co-ordinates.

Variation of a functional.Variational problems.Euler's equations and its various cases.Externals. Functional depending on n unknown functions, Functionals depending on higher derivatives. Variational problems in parametric form.Isoperimetric problem.Canonical form of Euler's equation.Functionals depending on functions of several independent variables.Ostrogradsky's equation.

Advanced Calculus

Functions of several variables, Contour lines, Level curves and level surfaces, Limits and continuity for functions of two and more variables, Partial derivatives, Partial derivatives and continuity, Differentiability, The chain rule for functions of two and three variables.

Directional derivatives, Gradient vectors, Tangent planes, Linearization and differentials, Extreme values and saddle points, Local and absolute maxima / minima, The method of Lagrange multipliers (with one constraint only), Taylor's series for function of two variables, Partial derivatives with constrained variables.

Double integral over a rectangle region, Double integral as volume, Double integral over non rectangular regions, Fubini's Theorem, Area of a region in a plane, Transformation of double integral from Cartesian to polar co-ordinates and vice-versa, Spherical and cylindrical co-ordinates, Triple integral in Cartesian, Cylindrical and spherical co-ordinates.

Line integral, Gradient field, Green's Theorem in a plane, Evaluation of line integral using Green's Theorem, Surface Area, Surface integral, Strokes Theorem.

Algebra

Binary operations, definitions of Group, Ring and Field with examples, vector spaces, subspaces, span of a set, sum of subspaces, linear dependence and independence, dimension and basis, coordinates of a vector relative to the ordered basis, dimension theorem.

Linear transformation and its properties, range and kernel of a linear transformation, rank and nullity of linear transformation, rank-nullity theorem, inverse of linear transformation.

The vector space L(U,V) and its dimension, composition of linear transformations, matrix associated with a linear transformation, linear transformation associated with a matrix, matrix as a linear transformation and its rank and nullity.

Elementary row operations and row- reduced echelon form, inverse of a matrix through elementary row operation, solution of a system of linear equations, eigen-values, eigen-vectors.

Real Analysis

Sets, Intervals: Open and closed, Bounded and unbounded sets: Supremum and infimum, Neighbourhood of a point, interior points, Open sets and results, Limits points of a set, Bolzano – Weierstrass Theorem, Closed sets and related results, Countable and uncountable sets, Compact sets and related results.

Limit of a function, Theorems on algebra of limits, Sequential approach, Cauchy's criteria for finite limits, Continuous functions, Discontinuous functions, Theorems on continuity, Properties of continuous functions on closed intervals, Uniform continuous functions and related results.

Definitions of derivatives and related results, Increasing and decreasing functions, Darboux's theorem, Rolle's theorem, mean value theorems of differential calculus and their applications, Taylor's theorem with various forms of remainder, Maculaurin's theorem, Taylor's infinite series, Maculaurin's infinite series expansion of some functions.

Functions of bounded variations and their properties, Variation function and related results, Jordon theorem, Vector valued functions, Vector valued functions of bounded variation and related results.

Concept of sequence, Limit points of a sequence, Limit inferior and superior, Convergent and divergent sequences with related theorems; Cauchy's general principle of convergence, Algebra of sequence and statement of related theorems with applications, Monotonic increasing and decreasing sequences, Several examples on this unit with solutions.

Introduction of infinite series, Sequence of partial sums and convergence of infinite series, Necessary condition for the convergence of an infinite series with proof, Positive term series, Comparison tests (first type and limit form), Cauchy root test, D'Alembert's ratio test with their applications, Alternating series, Leibnitz test, Absolute and conditional convergence, Series of arbitrary terms, Abel's and Dirichlet's tests.
Definition and existence of Riemann integral; Inequalities for Riemann integrals; Refinement of partitions; Darboux's theorem'; Theorems on conditions of integrability; Theorems on integrability of the sum, difference, quotient and product of integrable functions (without proof), Theorems on integrability of the modulus and square of integrable functions, The Riemann integral as a limit of sums, Theorems on integrable functions, The fundamental theorem: First and generalized first mean value; Theorems of integral calculus, Integartion by parts, Change of variables, Second mean value theorem.
Definition and existence of the integral, refinement of partitions, Condition of integrability and related results, Integral as a limit of sums and related results; Fourier series; Bessel's inequality; Dirichlet's criteria of convergence of Fourier series; Fourier series for even and odd functions; Fourier series on $[0,2\pi]$, $[-l,l]$ and $[0,l]$, where 1 is a real number.

Metric Spaces and Complex Analysis

Definition and examples of metric spaces; Statements of inequalities of Holder and Minkowski; Properties of metrics; Bounded and unbounded metric spaces; Distance between sets and diameter of a set; Open sets and interior points; Limit points, closed sets and closure of sets; Exterior points, boundary points, boundary of a set; Subspaces.
Convergent and Cauchy sequences in metric spaces; Complete metric spaces; Cantor's intersection theorem; Completion of a metric space; Basis of a metric space; First and second countable metric spaces; Dense sets; Separable metric spaces and examples; Nowhere dense sets; Baire's category theorem.
Open cover; Compact metric spaces; Finite Intersection property; Sequentially compact spaces; Wolzano-Weierstrass property; ε-net and totally bounded spaces; Lebesgue's covering lemma Continuous function between two metric spaces and its characterizations; Continuous functions and compact spaces; Uniform continuous functions; Homeomorphism and equivalent metrics; Uniform convergence sequences of functions; Cauchy Criterion for uniform convergence.
Revision of basic concepts; Triangle inequality; Roots of complex numbers; Region in the complex plane; Functions of a complex variable; Theorems on Limits; Limits involving the point at infinity; Continuity; Derivatives; Examples.
Cauchy-Riemann (CR) equations; Sufficient conditions for the differentiability; Polar form of CR equations; Analytic functions; Harmonic functions; Harmonic conjugate; Polar form of Laplace equation; Complex valued functions; Complex line integrals/contour integrals; Examples.
Cauchy-Goursat theorem (without proof); Consequences of Cauchy-Goursat theorem; Cauchy's integral formula; Cauchy's integral formula for higher order derivatives; Morera's theorem; Cauchy's inequality; Liouville's theorem; Fundamental theorem of algebra; Gauss' mean value theorem; Maximum and minimum modulus theorems/principles; Examples.
Power series; Taylor's series; Laurent's series; Singular points; Classifications of singularities; Zeros and poles of order m; Residues; Calculation of residues; Residue theorem; Evaluation of integrals; Examples.

Abstract Algebra

Binary relation, Function, Binary Operation, Groups, Various properties and examples of group, Subgroups, Properties of subgroups, Normal subgroups and important results, Cyclic groups, generator, Properties of Cyclic groups.
Cosets, Lagrange's theorem, Euler theorem, Fermat's theorem (with proofs), Isomorphism and homomorphism of groups and their examples and results, Quotient group.

First, Second and Third Isomorphism Theorems (with proofs), Direct product of groups and its related results.
Permutations, even and odd permutations, transportation, disjoint cycles, permutation groups and its related results, Cayley's theorem, Cauchy's theorem (with proofs)
Rings, Zero divisors, Integral domains, Division Rings, Field, Subrings and Ideals, Congruence modulo a subring relation in a ring, Simple ring, Algebra of ideals, Ideal generated by a subset, Nilpotent ideals, Nil ideals, Quotient rings, Prime and Maximal ideals.
Homomorphism in rings, Natural homomorphism, Kernel of a homomorphism, Fundamental theorem of homomorphism, First and second isomorphism theorems, Field of quotients, Embedding of rings, Ring of endomorphisms of an abelian group.
Factorization in integral domains, Prime and irreducible elements, H.C.F. and L.C.M. of two elements of a ring. Principal ideals domains, Euclidean domains, Uniquefactorisation domains, Different relations between principal ideal domains, Euclidean domains and unique and unique factorization domains.
Polynomials rings, Algebraic and transcendental elements over a ring, Factorization in polynomial ring R[x]. Division algorithm in R[x] where R is a commutative rings with identity, Properties of polynomial ring R[x] if R is a field or a U.F.D., Gauss lemma, Gauss Theorem (statement only), Eisenstein irreducibility criteria and its applications, Division algorithm for polynomial ring F[x], where F is a field, Reducibility test for polynomials of degree 2 and 3 in F[x].

Mechanics

Vector moment of a force, Varignon's Theorem on moments, Resultant of a couple, Resultant of coplanar forces, Equation of line of action of the resultant. Equilibrium of a rigid body, Conditions of equilibrium of three force body. Cables, Suspension and parabolic cables, Intrinsic and Cartesian equation of a catenary, Sag and span, maximum tension in a cable.
Virtual work.Principle of virtual work.Determination of tension in a string and thrust in a rod.Solutions of problems involving equilibrium by principle of virtual work. Centre of gravity. Determination of centre of gravity by integration. Centre of gravity of arcs, plane areas, enclosed areas, solids of revolution and surfaces of revolution.
Kinematics and kinetics, Tangential, normal, radial and transverse components of velocity and acceleration.Motion of projectile without resistance.Projectile motion up and down an inclined plane.Tangent problems.Motion in a resisting medium including projectiles.Upward and downward motion in a resisting medium.
Motion of particles in central orbits.Stability of circular orbits.Kepler's laws of planetary motion.Plane impulsive motion.Direct and oblique impact.Loss of energy during impact.Impact against a fixed plane.Simple harmonic motion.Motion of a simple pendulum.

Set Theory, Number Theory and Differential Geometry

Sets, Cartesian product of sets, Equivalence relation and partition, Fundamental theorem of equivalence relation, Functions, their restrictions and extensions, Characteristic functions and choice functions, Equivalent sets, Infinite sets, Denumerable sets, Countable sets, The continuum, Cardinal arithmetic, Inequalities of cardinal numbers, Cantor theorem.
Schroder Bernstein theorem, Continuum hypothesis, Partially ordered sets, Totally ordered sets, First and last elements, Maximal and minimal elements, Upper and lower bounds, Similar sets and ordered types, Well ordered sets, Transfinite nduction, Limit elements, Initial segments, Similarity between well ordered sets and their subsets, Ordinal numbers and their properties.
Thedivision algorithm and derived results, Least common multiple, Greatest common divisor, Euclid's algorithm, prime numbers and related results, Fundamental theorem of arithmetic, Relatively prime integers, Euler's function, Multiplicative arithmetical functions, Mobius functions, Mobius inversion formula.

Congruences, Euler's theorem, Fermat's theorem, order of an integer mod m, linear congruences, Chinese remainder Theorem, algebraic congruence mod p, Lagrange's theorem, Wilson's theorem, Algebraic congruence modulo a composite modulus.
Space curves. Examples. Plane curves. Parameterization of curves (Generalized and natural parameters) Change of parameter regular curves and singularities. Contact of curves.Contact of a curve and a plane.Frenettrihedron.Osculating plane.Serret-Fernet formulae.Involutes and Evaluates, Fundamental Theorem for space curves.
Surface in R^3.Implicit and explicit forms of the equation of a surface. Parametric curves on surfaces. Tangent plane.First fundamental form.Angle between two curves on a surface.Area of a surface.Invariance under co-ordinate transformation.
Second fundamental form on a surface, Gauss map and Gaussian curvature.Gauss and Weingrten formulae.Christoffle symbols. Some co-ordinate transformations.Goddazi equation and Gauss theorem.Fundamental Theorem of surface Theory.
Curvature of a curve on a surface.Geodesic curvature and normal curvature.Geodesics.Principal directions and lines of curvature.Rodrigue formula.Asymptotes Lines.Conjugate Directions.Developable surfaces.