DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
Syllabus for Ph.D. Admissions Test 2020-2021
SECTION-B

Calculus

Limit of a function, Continuous functions, Derivative of a function, Applications of derivatives, Rolle's Theorem, mean value theorem of differential calculus and their applications. Functions of several variables, level curves and level surface, partial derivatives and directional derivatives, linearization and differentials, extrema of two variables function, curl and divergence of a vector field. Double integral and triple integral, line integral, Green's theorem, surface area and Stokes theorem.

Set Theory and Number Theory

Countable and uncountable sets, Equivalence relation and partition, Fundamental theorem of equivalence relation, functions, their restrictions and extensions, Equivalent sets, Infinite sets, the continuum, Cardinals, Cardinal arithmetic, Inequalities and cardinal numbers, Cantor's theorem, Schroeder Bernstein theorem, Continuum hypothesis.
Ordinal numbers and their inequalities, Addition and multiplication of ordinals, Distributive laws, Structure of ordinal numbers, Axiom of choice, Zorn's lemma and well ordering principle, Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder theorem, Euler's Φ-function, primitive roots.

Linear Algebra

Vector space, Subspace, linear dependence, Basis and dimension, Linear transformation, Kernel of a transformation, Rank and nullity of transformation, Vector space L(U,V), Matrix associated with a linear transformation and vice-versa, rank and determinant of matrices, Eigen-values and Eigen- vectors, Diagonalization of Matrices, Cayley-Hamilton Theorem, Linear functional, Dual basis, Computing of a dual basis, Dual vector spaces, Annihilator, Second dual space, Dual transformations, Minimal Polynomial, Invertible Linear transformation, Invariant Subspaces, Canonical form, Jordan Form, Forms on vector spaces, Bilinear Functionals, Symmetric Bilinear Forms, Skew Symmetric Bilinear Forms, Rank of Bilinear Forms, Quadratic Forms, Classification of Real Quadratic forms.

Group Theory

Groups, Subgroups, Normal subgroups, Cyclic groups, Generators, Lagrange's theorem, Isomorphism of groups, Quotient groups, Fundamental theorems of homomorphism's in groups, Permutation, Permutation groups, Cayley's theorem, Relation of conjugacy, Conjugate classes of a group, Class equation in a finite group and related results, Partition of a positive integer, Conjugate classes in S_n, Sylow's theorems, External and internal direct products and related results. Commutator subgroup of a group, Subnormal series of a group, Refinement of a subnormal series, Length of a subnormal series, Solvable groups and related results, Nilpotent groups, relation between solvable and nilpotent groups, Composition series of a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups.

Advanced Ring Theory

Examples and fundamental properties of rings. Direct and discrete direct sum of rings. Ideals generated by subsets and their characterization in terms of elements of the ring under different conditions. Sum and direct sums of ideals, Ideals products and nilpotent ideals, Minimal ideals, Complete matrix ring, Ideals in complete matrix ring, Residue class ring. Homomorphisms, Subdirect sum of rings and its characterization, Zorn's Lemma, Subdirectly irreducible rings, Boolean rings, Prime ideals and m-systems. Different equivalent formulation of prime ideals. Semiprime ideals and n-systems, equivalent formulation of semiprime ideals, Necessary and sufficient conditions for an ideal to be a prime and semiprime ideal, Prime radical of a ring, Prime rings and its characterization in terms of prime ideals. Primeness of complete matrix rings. D.C.C. for ideals and the prime radical, Jacobson radical and simple properties, Relationship between Jacobson radical and prime radical of a ring.

Field Theory

Field Extensions, Algebraic extensions, Splitting fields, Normal extensions, Multiple roots, Finite fields and separable extensions, Automorphism groups ad fixed fields, Fundamental Theorems of Galois Theory, Fundamental Theorem of Algebra. Einstein's criterion, Gauss theorem.

Real Analysis

Limit of a function, sequential approach, Cauchy's criteria for finite limits, Continuous and discontinuous functions and their properties, Uniform continuous functions, Increasing and decreasing functions, Darboux's theorem, Taylor's theorem, Maculaurin's theorem, Taylor's and Maculaurin's infinite series with applications, Functions of bounded variations and their properties, Variation of functions, Jordon theorem. Limit points of a sequence, Types of limits, Convergent and divergent sequences and related results, Cauchy's general principle of convergence. Algebra of sequences and related results with applications, Monotonic increasing and decreasing sequences, Infinite series, Convergence of infinite series, Positive term series, various type of tests for convergence of series, Alternating series, Leibnitz test, absolute and conditional convergence, Series of arbitrary terms, Abel's and Dirichlet's tests. Riemann integral, Darboux's theorem, conditions of integrability, Mean value theorems of integral calculus, Change of variables, Fourier series, Fourier series for even and odd functions, Sequences and series of functions, pointwise and uniform convergence, tests for uniform convergence: Cauchy criterion, Weierstras's M-test, Abel's and Dirichilet's test, uniform convergence and continuity, uniform convergence and differentiation, uniform convergence and integration, Weierstras's approximation theorem, existence and properties of Riemann-Stieljes integrals, intergration of vector valued functions, rectifiable curves, power series, uniqueness theorem, Abel's and Taylor's theorems, rearrangement of terms of series and related results, inverse function theorem and implicit function theorem.

Metric Spaces

Metric spaces, Properties of metrics, Distance between sets, Limit points, Closed sets, Exterior points, Boundary points Boundary of a set, Subspaces, convergent and Cauchy sequences, Complete metric spaces, Cantor's intersection theorem, Completion of a metric space, Countable metric spaces, Separable metric spaces, nowhere dense sets, Baire's category theorem, Compact metric spaces and related results, Sequentially compact spaces and related results, Continuous functions their characterizations, Continuous functions on compact spaces, Uniform continuous functions, Homeomorphism and equivalent metrics.

Complex Analysis

Functions of a Complex variable, Limits, Continuity and Derivatives of functions, Cauchy-Riemann equations and related results, Analytic functions, Harmonic functions, Harmonic conjugate, complex valued functions, Complex line integrals and contour integrals, power series, Taylor's series: Laurent's series, fundamental theorem of line integrals (or contour integration), Cauchy's theorem, Cauchy-Goursat theorem, symmetric, Cauchy's theorem for a disk, Cauchy's integral theorem, index of a closed curve, advanced versions of Cauchy integral formula with applications, Cauchy's estimate, Morera's theorem, Convergence of sequences and series of complex valued functions, Weierstrass' M-test, power series as an analytic function, root and ratio test, uniqueness theorem for power series, zeros of analytic functions, identity theorem and related results, maximum/minimum modulus principles and theorems, Schwarz' lemma and its consequences, isolated and non-isolated singularities, removable singularities, poles, characterization of singularities through Laurent's series, Calculus of residues, residue at a finite point, results for computing residues, residue at the point at infinity, Cauchy's residue theorem, residue formula, meromorphic functions, number of zeros and poles, argument principle, evaluation of integrals, Rouche's theorem, Mittag-Leffer expansion theorem and their applications.

Conformal mappings, basic properties of Möbius maps, images of circles and lines under Mobius maps, fixed points, triples to triples under Möbius maps, cross-ratio and its invariance property, mappings of half-planes onto disks, Inverse function theorem and related results.

O.D.E. and P.D.E.

Linear differential equations of order n, complementary functions and particular functions, Cauchy-Euler's and Legendre linear equations, series solution of differential equations with variable coefficients, Initial value problems and boundary value problems, linear dependence and independence of solutions, equations with constant as well as variable coefficients, Wronskian, variation of parameter, method of undetermined coefficients, reduction of the order of an equation, method of Laplace's transform, Lipschitz's condition and GronWall's inequality, Picards theorems, dependence of solution on initial conditions and on the function, Continuation of solutions, Non-local existence of solutions Systems as vector equations, existence and uniqueness of solutions, Strum-Liouvilles system, Green's function and its applications to boundary value problems, some oscillation theorems such as Strum theorem, Strum comparison theorem and related results, System of first order matrix equation, fundamental matrix, Non-homogeneous linear system, Linear system's with constant as well as periodic coefficients.

Classifications of first order pde's, Lagrange's equations, Non-linear pde of first order and their different forms, Charpit's method, Homogeneous linear pde'sof higher order with constant coefficients, Different cases for complimentary functions and particular integrals, Non-homogeneous pde's of higher order with constant coefficients, Classification of second order pde's, Method of separation of variables for Laplace, Heat and Wave equations.

Measure Theory

Measurable and non-measurable sets, measurable functions, BorelLebesguemeasurability, measure and outer measure, extensions of a measure, uniqueness of extension, completion of measure, integration of non-negative functions, Riemann and Lebesgue integrals, the four derivatives, Lebesgue differentiation, differentiation and integration, measure spaces, convergence in measure.

Functional Analysis

Normed spaces, Banach spaces and their properties, closedness, completion of normed linear spaces, Finite dimensional normed spaces and subspaces, equivalent norms, compactness and finite dimension, Riesz's lemma, quotient spaces, Bounded linear operators with their norms and properties, algebraic and topological dual spaces, examples and properties of dual spaces, weak convergence and strong convergence, reflexive normed spaces, Hahn-Banach theorems and their consequences (analytic and geometric forms), pointwise and uniform boundedness, uniform boundedness principle and its applications, open and closed maps, open mapping and closed graph theorems, their consequences and applications, Banach contraction theorem with its applications, Inner product space, parallelogram law, polarization identity and related results, Schwartz inequality, Hilbert space, orthogonality of vectors, orthogonal complements, projection theorem and related results, orthonormal sets and sequences, Bessel's inequality, total orthonormal sets, Parseval formula, separable Hilbert spaces, Riesz representation theorems for bounded linear functional and bounded sesquilinear form, Hilbert adjoint operators, self adjoint operators, unitary and normal operators.

Topology

Definitions and examples of topological spaces, basic definitions, Bases and subbases, Topology generated by the subbasis, subspaces and relative topology, Continuous functions and homeomorphism., first and second countable spaces, Lindelof spaces, Separable spaces, Second countability and separability, Separation axioms, T_0, T_1, T_2, T_3, T_4 spaces and their characterizations and basic properties, Urysohn's lemma, Tietze extension theorem.
Compact spaces and their basic properties, Separation of a space, Connected spaces, Connected sets in the real line, Totally disconnected spaces, Intermediate value theorem, path connected, Components, Path components, Locally connected spaces, Locally path connected spaces, Totally disconnected spaces, Continuous functions and connected sets.
Product topology (finite and infinite number of spaces), Tychonoff product topology in terms of standard sub-base and its characterizations, Projection maps, Separation axioms and product spaces, Connectedness and product spaces, Compactness and product spaces (Tychonoff's theorem), Countability and product spaces.

Differential Geometry

Curves, curvature and torsion, Seret-Frenet Formulae, fundamental theorem of curves, surfaces, first and second fundamental forms, Gaussian curvature, curves on surfaces, geodesic, normal curvatures, principal curvature, Charts, Atlases, Manifolds, Differentiable structure on a manifold, Smooth maps, Tangent vectors and Tangent space, Vector fields, Lie product of Vector fields, Jacobian of a smooth map, Integral curves on a manifold, Cotangent spaces, pullback of l-forms, Tensor fields, Differential forms, Exterior product and derivative, Connexion, parallelism, Geodesic, Covariant differentiation, Torsion, Curvature, Structure equation of Cartan.